Trumpet with near-perfect harmonicity: Design and acoustic results

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This paper presents a mathematical design methodology for determining the shape of a trumpet air column that has near-perfect harmonicity, whose components are discontinuity-free, and whose input impedance peak heights are balanced over the playing range. The simulation model employed assumes linear wave propagation and uses cylindrical element discretization with a plane wave approximation. Acoustic measurements are made using a test set-up with an estimated relative measurement error of ±3 cents. Comparisons of measured results are given for the presented design (Macaluso trumpet) and the same trumpet air column with the bell replaced by a commercially used generic trumpet bell of unknown shape (Generic trumpet). For acoustic resonance modes 2–13 (233–1515 Hz), the measured root-mean-square (rms) harmonicity deviation is 5 cents for the Macaluso trumpet, whereas it is 18 cents for the Generic trumpet. However, considering the estimated measurement uncertainty, each of those deviations is somewhat over-stated. For that same range of resonances, the rms deviation between measured and calculated resonance frequencies for the Macaluso trumpet is 3 cents, thus validating the presented simulation model and equations.
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I. INTRODUCTION

While there has been considerable advancement in acoustical analysis methods, the trumpet designer is still left with the intractable problem of determining an air column shape that satisfies specific acoustic criteria. That vexing conundrum is addressed in this paper.

The objectives established for this paper are as follows: (1) to develop a design methodology for determining an air column shape based upon selected acoustic criteria; (2) to design a trumpet air column using such methodology; (3) to precisely replicate such design in a test instrument; (4) to make accurate acoustic measurements for the test instrument; (5) to compare acoustic measurements for the test instrument (Macaluso trumpet) and the test instrument with the bell replaced by a commercially used generic trumpet bell of unknown shape (Generic trumpet); (6) to demonstrate that theoretical calculations for the Macaluso trumpet agree closely with measurements.

These topics are presented in the following sections of this paper: the acoustic criteria selected for design, the wall shapes used, the simulation model and the equations used, the design procedure, the means by which the design shape is precisely replicated in the test trumpet, a comparison of the methodology to a published means of computer optimization, a new set-up for making accurate acoustic measurements, comparisons of measurements that demonstrate the harmonicity superiority of the Macaluso trumpet over the Generic trumpet, comparisons of measurements and calculations that validate the simulation model, and the need for future playing tests. A photograph of the test chamber is shown in Fig. 1.

II. ACOUSTIC CRITERIA SELECTED FOR DESIGN

Three well-known acoustic criteria are selected for the design of the trumpet.

(1) *Discontinuity-free:* Benade[1] asserts (p. 425), "Trouble can be caused by a small change in cross section, a sharp bend, or an ill-chosen change in taper. Such discontinuities return a pre-mature echo of significant size to the mouthpiece, an echo that is not even a replica of the original disturbance. Such ill-timed, ill-shaped return echoes can upset the best-trained of lips, and, having spoiled the steadiness of their initial vibration, will ruin the attack."
(2) *Near-perfect harmonicity:* Benade[1] presents a formal definition of a regime of oscillation and affirms that (p. 400), "the more resonances that are present to cooperate and the more accurately these are aligned, the easier it is to play the notes." Fletcher and Rossing[2] (F&R) state that (p. 451), "… for a well-tuned horn having many harmonically related resonances, … reflection phase shifts at the open bell vary with frequency in such a way as to cancel the transit time dispersion and generate a coherent reflected pulse at the instrument mouthpiece. This may be one of the major features distinguishing a really good instrument."

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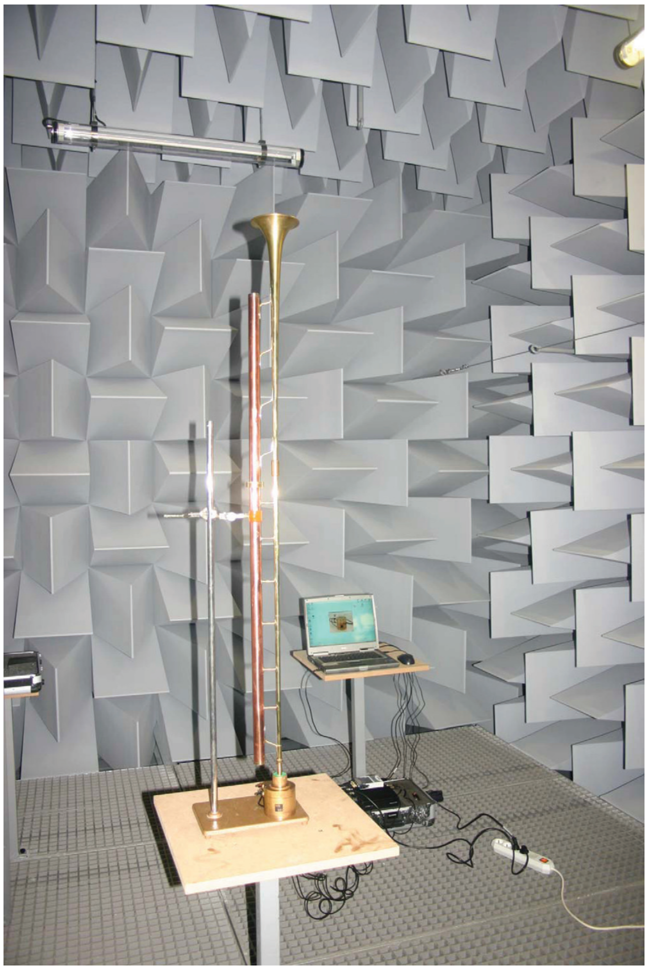

FIG. 1. (Color online) Photograph of test chamber.

(3) *Balance of resonance peak heights:* F&R[2] provide a criterion (p. 436): "…important is the balance of resonance peak heights over the playing range, since easy transition from one resonance to another is essential if the instrument is to be musically flexible."

The acoustic criteria given above have been known for many years. More recently, some musical acoustics researchers have questioned the use of such objective acoustic criteria.[3–5] However, an air column design must be based upon selected acoustic criteria. The authors embrace these particular criteria and choose to apply them for design purposes, leaving any challenge of such criteria to others. While these criteria are considered to be essential, other criteria are not excluded from use with this methodology.

III. DESCRIPTION OF WALL SHAPES USED

The major components of a trumpet air column are the mouthpiece, leadpipe (or mouthpipe), cylindrical tubing, and bell. The principal dimensions of a component are length and inside diameter at each end. Such component dimensions vary among makers, with no major differences in musical performance. Any values of such dimensions may be used with the design methodology presented. The principal dimensions of such components used in the Macaluso trumpet are similar to those found in selected professional model Bb trumpets, as given below:

The mouthpiece is 8.89 cm long with an exit inside diameter of 8.550 mm. The leadpipe is 31.75 cm long, tapering from 8.763 to 11.684 mm inside diameter. The cylindrical tubing is 39.90 cm long and 11.684 mm in inside diameter. The bell is 60.96 cm long, flaring from 11.684 to 122.24 mm inside diameter.

The design is based upon circular cross sections, a straight-axis, and no valves. The test trumpet is built in accordance with the design. Commercial trumpets are constructed with bends for reasons of portability. The principal acoustic effects of a bend vary with the sharpness of the bend, resulting in an increase in velocity of sound and a decrease in impedance.[1,2] Any increase in velocity of sound within a bend shortens the bend acoustic length, which is compensated in the instrument design.

To avoid Benade's "ill-chosen change in taper" and resultant discontinuities, all wall shapes used are mathematically continuous. One such essential continuous wall shape used in the design is referred to as a mathematical transitional wall shape, and it is described below.

Mathematical transitional wall shapes are used between cylindrical and conical wall shapes in designing the mouthpiece and the leadpipe. At each end of the transitional wall shape, the values of the radius, and the first and second derivatives of radius, exactly match those of the adjoining cylindrical and conical shapes. The radius of the transitional wall shape as a function of length is defined by the first five terms of a power series. The unknown coefficients of such a series are found from the simultaneous solution of the equations for the radius, and the first and second derivatives of radius. The second derivative is zero at each end of the transition and it varies continuously over the length, attaining a maximum or minimum value at the mid-length.

The mouthpiece and the leadpipe used in the test trumpet employ the mathematical transitional wall shapes described above. Such mouthpieces and the leadpipe had been previously built as components of professional trumpeters' instruments, with favorable results.

The continuous mouthpiece wall shape is described by ten equations using 21 design parameters. The mouthpiece significantly affects the input impedance and the harmonicity of the instrument.

When the trumpet is played, the player's lips penetrate into the mouthpiece cup. The magnitude and detailed configuration of such lip penetration vary with the player and the mouthpiece. A first-order acoustic effect of lip penetration is the displacement of mouthpiece cup volume. To quantify the lip penetration effect in acoustic simulations, a simplifying assumption is made that the lips penetrate into the cup to a plane parallel to the mouthpiece rim plane.

All harmonics of all fundamental tones can resonate in an air column with perfect harmonicity only when each fundamental tone is produced using the same value of average lip penetration. Accordingly, constant average lip penetration is used in the design.

For the mouthpiece used in the trumpet design, the simulated effect of mouthpiece average lip penetration on

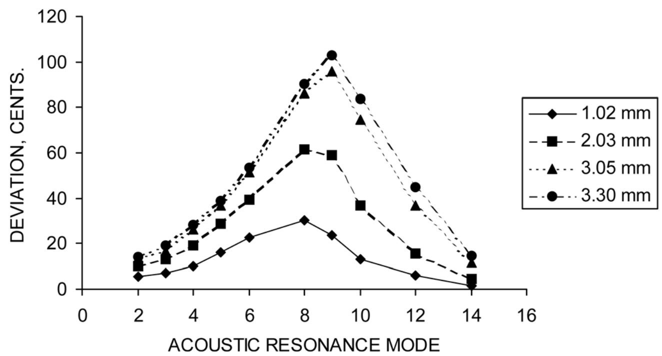

FIG. 2. Simulated resonance frequency deviations produced by various values of average lip penetration.

resonances is shown in Fig. 2 (the simulation model and equations used are described in Sec. IV). The magnitude of the lip penetration effect is very significant. However, the consequence of lip penetration is not widely appreciated.

To conform to commercial trumpet construction, a gap between the mouthpiece and the leadpipe and a simulated extended tuning slide (with its two gaps) are included in the straight test trumpet. A gap is a short cylinder with an inside diameter larger than the adjoining components. The gap diameters, lengths, and axial positions within the air column are typical of commercial trumpet construction. Those gaps produce the sole discontinuities in the air column design.

A well-known shape that has been used in the design of trumpet bells is a Bessel horn (BH). Benade[1] (p. 408) stated that, "the bell parts of trumpets and trombones have been made in shapes that correspond closely (but not exactly) to the shapes of Bessel Horns having values of the flare exponent, m, between about 0.50 and 0.65." "Their usefulness is greatly enhanced by the fact that such shapes correspond closely enough to what has proved musically serviceable to permit quite passable instruments to be made from them."

However, when a BH is used as an approximation of a trumpet bell shape, the BH hyperbolic shape produces a discontinuity at the small-end, where it is connected to cylindrical tubing. A modified Bessel horn (MBH) bell equation is developed to eliminate such a discontinuity:

$$r = \frac{K}{(x+b)^m} - A\sin\left(\frac{x}{L}\pi\right)\sin\left(\frac{x}{L}\frac{\pi}{2}\right) - B\sin\left(\frac{L-x}{L}\pi\right)\sin\left(\frac{L-x}{L}\frac{\pi}{2}\right), \quad (1)$$

where r is the MBH bell radius, as a function of axial coordinate, x (x is zero at the large-end and extends to L). By specifying values of the bell length, L, the bell large-end radius, r_l, the bell small-end radius, r_s, and the constant, b, the constants m and K are determined from Eqs. (2) and (3). Both the first and second derivatives of radius at the bell small-end are equal to zero when the coefficients A and B have the values given by Eqs. (4) and (5).

$$m = \frac{\ln\left(\frac{r_l}{r_s}\right)}{\ln\left(\frac{L}{b}+1\right)}, \quad (2)$$

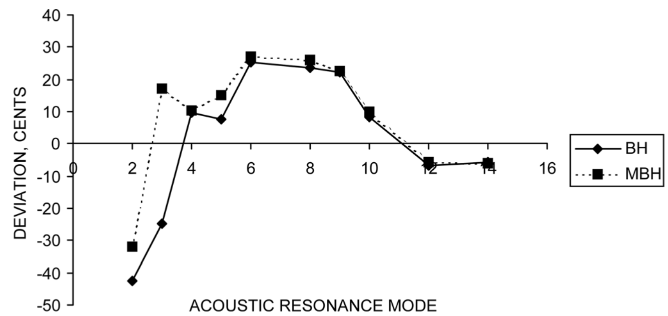

FIG. 3. Simulated harmonicity deviations for trumpets using BH and MBH bells.

$$K = r_l b^m, \quad (3)$$

$$A = \frac{L}{\pi}\frac{mK}{(L+b)^{m+1}}, \quad (4)$$

$$B = \left(\frac{L}{\pi}\right)^2 \frac{m(m+1)K}{(L+b)^{m+2}}. \quad (5)$$

The MBH bell eliminates the discontinuity at the small-end. However, simulations of the previously described air column using a MBH bell and a BH bell both exhibit relatively large harmonicity deviations (see Fig. 3). It is desired to reduce the harmonicity deviations for each resonance to within the hearing frequency detection threshold of 3 cents.[6] The MBH bell shape must be mathematically "corrected" to reduce the harmonicity deviations. The required bell shape is a yet unknown continuous shape that yields near-perfect harmonicity.

IV. THE SIMULATION MODEL

A very accurate simulation model is required to design a trumpet with near-perfect harmonicity.

A. Description

The simulation model used assumes the wave propagation to be linear, one-dimensional, and plane. The propagation number, viscothermal losses, and a radiation impedance model are included as complex quantities. The input impedance is computed using finite cylindrical element discretization. In the limit in which the length of the element becomes infinitesimal, for the plane wave assumption and an air column of small conical angle, the representation is exact.[2] However, the rapidly increasing conical angle of the trumpet bell flare leads to uncertainty. Nevertheless, for a typical trumpet, such a model has been reported to be adequate for up to about 1500 Hz.[7]

The complex values of the propagation number and viscothermal wall losses are evaluated at the cylindrical element radius. Caussé *et al.*[8] published a comparison of measured and computed resonance frequencies for long cylindrical tubes, in which viscous and thermal wall loss effects are predominant. Caussé *et al.* found that the root-mean-square (rms) uncertainty in computed vs measured frequencies was less than one cent over a frequency range of 400–3200 Hz. Therefore, the uncertainty in resonance frequencies resulting from using such viscothermal losses in the simulation model is very small.

The radiation impedance from the large open end of the bell flare to the surrounding space is a major source of uncertainty in modeling. In the design of the test trumpet, the radiation impedance was taken as the average of the radiation impedance for flanged pipe and open pipe, given by F&R[2] and Caussé *et al.*,[8] respectively. After the trumpet was produced, Silva *et al.*,[9] published a paper that presented improved radiation impedance equations for flanged pipe and open pipe. Simulations using both of those cases were evaluated, from which it was determined that the flanged pipe model produced slightly better agreement with measurements. Accordingly, the flanged pipe model of Silva *et al.* is used in the theoretical calculation model. While neither case is directly applicable to a trumpet bell, the flanged pipe model provides an approximation that agrees closely with measured results, within frequency limits.

Multimodal methods use a large number of axisymmetric wave propagation modes to determine the specific wave shape.[10,11] Such methods are the means by which the wave shape and the radiation (or reflectance) are accurately determined. However, such methods are mathematically difficult and they are not easily integrated into an inverse design methodology (where the unknown bell shape is yet to be determined).

The model used assumes that the trumpet air column has circular cross sections, a straight-axis, and no valves (the same as the test trumpet). The air column shape, including the mouthpiece, is described by a total of 62 design parameters in 35 equations (17 equations describe the bell). The model assumes the air column to be closed within the mouthpiece cup at a plane parallel to the mouthpiece rim plane.

A cylindrical element program with 0.25 mm length elements is used, except in the mouthpiece where 0.025 mm elements are used. Those element lengths yield resonance frequencies equal to the asymptotic element values.

B. Equations used

The radiation impedance for an infinitely flanged pipe, as given by equations (3), (21), and (22), respectively, by Silva *et al.*,[9] is as follows:

$$Z_r = -i\tan\left(k_0 L_0 - i\frac{1}{2}\ln(|R|)\right), \tag{6}$$

$$|R| = \frac{1+0.730(k_0a_0)^2}{1+1.730(k_0a_0)^2+0.372(k_0a_0)^4+0.0231(k_0a_0)^6}, \tag{7}$$

$$L_0 = 0.82159a_0 \times \frac{1+0.244(k_0a_0)^2}{1+0.723(k_0a_0)^2-0.0198(k_0a_0)^4+0.00366(k_0a_0)^6}, \tag{8}$$

where Z_r, k_o, L_o, R, and a_o denote the radiation impedance, the non-complex wave number, the end correction due to radiation, the reflection coefficient, and the inner radius of the large-end of the trumpet flare, respectively.

The ratio of cylindrical element radius to viscous boundary layer thickness and the ratio of cylindrical element radius to thermal boundary layer thickness, as given by the (F&R)[2] equations 8.12 and 8.13, respectively, are as follows:

$$r_v = 632.8af^{0.5}(1-0.0029\Delta T), \tag{9}$$

$$r_t = 532.8af^{0.5}(1-0.0031\Delta T), \tag{10}$$

where r_v, r_t, a, f, and ΔT denote the ratio of the cylindrical element radius to the viscous boundary layer thickness, the ratio of the cylindrical element radius to the thermal boundary layer thickness, cylindrical element radius, the frequency in hertz, and the temperature deviation from 300 K, respectively.

The cylindrical element complex wave number, as given by the F&R[2] equations 8.14 and 8.15, respectively, is as follows:

$$k = \frac{\omega}{c\left[1-\frac{1}{r_v\sqrt{2}}-\frac{(\gamma-1)}{r_t\sqrt{2}}\right]} - i\frac{\omega}{c}\left[\frac{1}{r_v\sqrt{2}}+\left(\frac{\gamma-1}{r_t\sqrt{2}}\right)\right], \tag{11}$$

where k, ω, c, and γ denote the complex wave number, the angular frequency, the velocity of sound in air, and the ratio of the specific heats of air, respectively.

The cylindrical element input impedance, as given by the F&R[2] equation 8.23, is as follows:

$$Z_{\text{in}} = Z_0\left[\frac{Z_L\ \cos(kL)+iZ_0\ \sin(kL)}{iZ_L\ \sin(kL)+Z_0\ \cos(kL)}\right], \tag{12}$$

where Z_{in}, Z_o, Z_L, and L denote the element input impedance, the element characteristic impedance, the element terminating impedance, and the element length, respectively.

V. THE DESIGN PROCEDURE

A. Perturbing the bell wall shape

To correct the bell shape, and thereby realize near-perfect harmonicity, multiple simultaneous perturbations of the initial MBH bell shape are used. Such a radial perturbation function is

$$r = \sum_{n=1}^{p} C_n\ \sin\left(\frac{l_n - x}{l_n}\pi\right)\ \sin\left(\frac{l_n - x}{l_n}\frac{\pi}{2}\right), \tag{13}$$

where r is the radius of the bell shape perturbation, as a function of axial coordinate, x. C_n is the perturbation coefficient (the coefficient is positive or negative), and l_n is the perturbation length (l_n is less than the bell length). The axial coordinate, x, is zero at the large-end of the bell and extends to l_n. The sum of a large number, p, of such perturbation functions (each with a different length and a different coefficient) is added to the MBH bell equation to produce a corrected bell shape.

B. Individual perturbations

The effect of an individual bell wall perturbation on resonance frequency is determined from comparative air

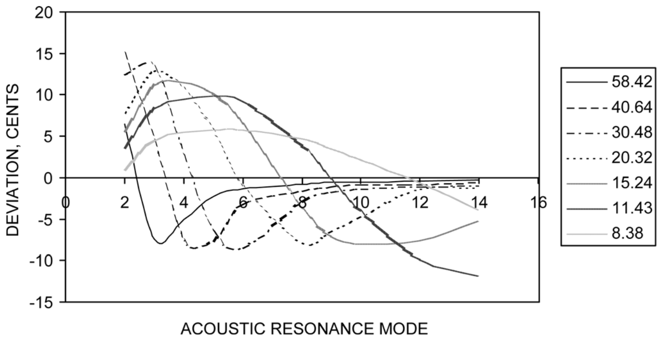

FIG. 4. Simulated resonance frequency deviations caused by bell perturbations. See Table I for perturbation lengths and the coefficients used.

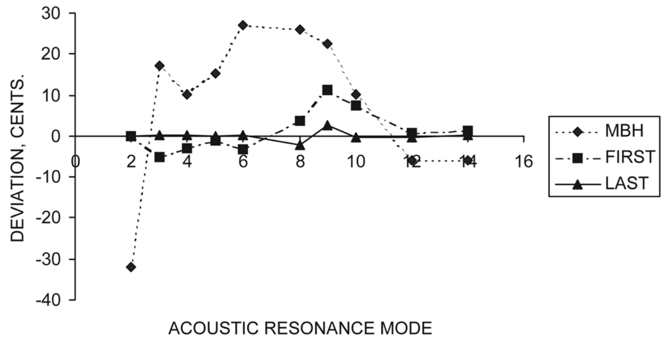

FIG. 5. Simulated harmonicity deviations for MBH bell and the first and last iterative shape corrections.

column simulations. Figure 4 shows the effects of selected individual bell wall shape perturbations on resonance frequencies [see Eq. (13)]. The arbitrary perturbation lengths and the positive coefficients used in Fig. 4 are shown in Table I. However, the coefficients used for design may be positive or negative. Perturbation lengths are measured from the large-end of the bell toward the small-end. Longer perturbations primarily affect only the lower acoustic resonance modes, whereas shorter perturbation lengths affect all modes. Near-perfect harmonicity is attained by the simultaneous use of 14 perturbations.

For purposes of illustration, a perturbation function having a length of 15.24 cm and a small positive coefficient is selected from Fig. 4. The frequencies of modes 2–7 are increased while the frequencies of modes 8–14 are decreased.

C. Iterative means of correcting the bell shape

Based upon the calculus, the effects on resonances of perturbations with infinitesimally small coefficients are assumed to be linear, symmetrical, and cumulative. Using the effects of individual perturbations (see Fig. 4), with coefficients to be specified, a computer program is used to calculate the linear, symmetrical, and cumulative effect of 14 simultaneous perturbations on resonances 2–14.

The design objective is to reduce the harmonicity deviations for the 13 resonances. A relatively simple non-linear algorithm is used to determine a set of coefficients that meets that objective.

The assumptions for infinitesimally small perturbations of the coefficients are not valid for large coefficients. Therefore, the coefficients required to obtain near-perfect harmonicity must be found by iteration. Starting with the harmonicity deviations produced by a MBH bell, an initial tentative set of perturbation coefficients required to partially correct such deviations is determined using the computer program.

Using those coefficients to define a partially corrected bell, a set of smaller harmonicity deviations is found from a simulation. Using the computer program, a set of incremental coefficients is found to further reduce the smaller deviations. Those incremental coefficients are algebraically added to the initial coefficients. The final set of corrected coefficients is found by repeating such an iterative process several times.

As the deviations are progressively reduced to smaller values, the required incremental changes are progressively reduced, and the predictive properties of the program become progressively more accurate. As few as four or five such iterations are required to obtain near-perfect harmonicity. Figure 5 shows the harmonicity deviations for an uncorrected MBH bell shape, and for the first and the last corrected bell shapes.

A second computer program is used to determine the cumulative effect of the 14 perturbations on the bell coordinates.

D. Adjusting the length of the air column

In using the previously described computer program, values of perturbation coefficients are determined to reduce the harmonicity deviations. The deviations (in cents) may be further reduced by adjusting them by a constant (in cents). Such an adjustment simulates a small change in air column length. The air column length can be subsequently revised to eliminate or reduce such an adjustment. For a Bb trumpet, a 0.25 mm change in the length of the cylindrical tubing changes the pitch by about 0.3 cent.

E. Effect of bell flare exponent

The important effect of the MBH bell flare exponent, *m*, on the input impedance envelope is determined by examining the envelopes of several air column designs. For each such air column design, the corrected bell shape has near-perfect harmonicity and a different value of the flare exponent. There is no optimum value for the flare exponent, but rather there is a difference in the balance of peak heights. An appropriate value of the flare exponent can thereby be selected, resulting in both near-perfect harmonicity and balanced peak heights over the playing range (the third acoustic criterion).

TABLE I. Perturbation lengths and coefficients.

Perturbation lengths (cm)	Positive coefficients shown in Fig. 4
58.42	0.016
40.64	0.030
30.48	0.042
20.32	0.060
15.24	0.090
11.43	0.120
8.38	0.110

F. The resulting trumpet design

The calculated harmonicity deviations for the resulting trumpet design are shown in Fig. 5, as the last corrective iteration. The calculated input impedance is shown in Fig. 11.

Except for the mouthpiece gap and the tuning slide gaps used to conform to commercial trumpet construction practice, the resulting air column is discontinuity-free, near-perfect harmonicity is attained, and the input impedance peak heights are balanced over the playing range.

As explained earlier, the simulation model used in designing the Macaluso trumpet differed slightly from the preferred theoretical model presented in Sec. IV. However, the difference between the model used in the design and the preferred model produced differences in resonance frequencies of only a few cents.

G. Manufacture of test trumpet

The same equations are used to manufacture the mouthpiece, leadpipe, tubing, and bell as are used to describe those shapes in the simulation model. The mouthpiece, mouthpiece receiver (a component which retains and accurately positions the mouthpiece in proximity to the leadpipe), and the mandrels for the leadpipe, tubing, and the bell are machined on computerized numerical controlled (CNC) machines to tolerances of ± 0.008 mm, or less. Extreme care is used to achieve concentricity in the assembly of the components.

VI. THIS DESIGN METHODOLOGY COMPARED WITH COMPUTER OPTIMIZATION

"Computer optimization of brass instruments" is described in a paper of that title,[7] and in related papers.[12–14] The paper[7] depicts a comprehensive method, which uses a robust computer program and a powerful mathematical algorithm in seeking an optimum solution. The paper presents an optimized trumpet wall shape that is derived from an input impedance target.

The objective for the methodology of this paper is similar to that of the above paper, inasmuch as both methods seek to derive a wall shape from specified acoustic criteria. However, it differs importantly in certain essential details. In this methodology, all wall shapes used are mathematically continuous, thereby precluding discontinuities and assuring feasibility of conventional manufacturing methods. Inasmuch as the professional trumpeter mouthpiece design is deemed inviolable, the mouthpiece is excluded from any change and the crucial mouthpiece shapes are precisely replicated using ten equations, 21 parameters, and 3500 coordinates. The continuous leadpipe and bell wall shapes are described by equations and use 1250 and 2400 coordinates, respectively. The leadpipe and the principal dimensions of the mouthpiece gap, tuning slide gaps, and cylindrical tubing are also excluded from any change. The bell shape is corrected to attain near-perfect harmonicity using 14 continuous radial perturbation functions to reduce harmonicity deviations. Near-perfect harmonicity for acoustic resonance modes 2–13 is achieved in as few as 4 or 5 rapidly converging iterations. The simulation model used is accurate to within 3 cents rms. The bell flare exponent is systematically varied to obtain a desired balance of input impedance peak heights as well as near-perfect harmonicity. All three selected acoustic criteria are completely satisfied.

VII. INPUT IMPEDANCE MEASUREMENT SET-UP

For the input impedance and resonance frequency measurements, a new set-up developed jointly by CTTM (Centre de Transfert de Technologie du Mans, 20 rue Thalès de Milet, 72000 Le Mans, France) and LAUM (Laboratoire d'Acoustique de l'Université du Maine, UMR CNRS 6613, Avenue Olivier Messiaen, 72085 Le Mans Cedex 9, France) is used.[15] This set-up is described in Sec. VII A. The measurement set-up allows the determination of the resonance frequencies with an uncertainty sufficient to significantly compare measurements between the two trumpets and to compare measurements with calculations.

A. Principle

Various input impedance measurement set-ups have been proposed in the literature.[16,17] They mainly differ in the means by which the volume velocity is determined. Currently, the solutions using one or more pairs of microphones are the most popular.[18,19] However, these are not well adapted to highly resonant systems such as wind instruments.

The impedance measurement set-up used is based upon a principle in which the pressure in the back cavity of the source is measured by a microphone. This microphone gives an estimation of the volume velocity of the source. In the set-up used, the source is a piezoelectric buzzer similar to that used in the set-up proposed by Benade and Ibisi.[16] The difference is that in the measurement set-up used, a small closed cavity in which a microphone measures the pressure, p_1, is connected to the back of the buzzer. The measured pipe is connected to the front of the buzzer *via* a small open cavity in which a second microphone measures the pressure p_2. See Fig. 6.

The pressure in the back cavity is at first order proportional to the flow U delivered by the source, and the pressure in the front open cavity is at first order equal to the pressure at the input of the pipe. The input impedance of the pipe $Z = p/U$ is thus at first order proportional to the transfer function between the two microphones. At first order, it can be written as,

$$\frac{p_1}{p_2} = -jC\omega Z, \tag{14}$$

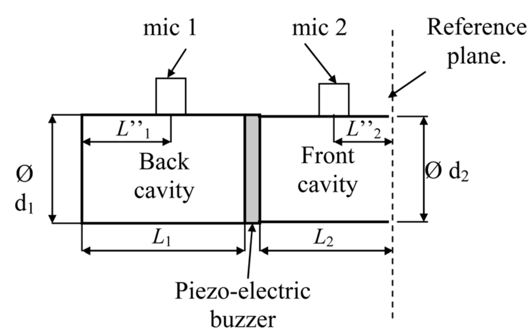

FIG. 6. Schematic drawing of the impedance measurement set-up and notations.

where $C = V/\rho c^2$ is the acoustic compliance of the back cavity of volume V, with ρ the air density and c the speed of sound.

In practice, Eq. (14) is not sufficient and is only valid for low frequencies. Moreover, it is necessary to take the relative sensitivity of the two microphones into account, since the measured transfer function is $H_{12} = (p_2/p_1)(s_2/s_1)$ with s_1 and s_2 being the respective sensitivities of microphones 1 and 2. However, it is possible to calculate more precisely the expression of the impedance by taking into account the geometrical dimensions of the sensor:

$$H_{12} = K\frac{Z+\beta}{1+\delta Z}, \quad (15)$$

where $K = -j\frac{1}{Z_{c1}}\frac{s_2}{s_1}\frac{\sin(kL_1)}{\cos(kL_1'')}\frac{\cos(kL_2'')}{\cos(kL_2)}$,

$$\beta = jZ_{c2}\tan(kL_2''),$$

$$\delta = j\tan(kL_2)/Z_{c2}.$$

Lengths L_1, L_2, L_1'', and L_2'' are dimensions related to the set-up and to the position of the microphones as indicated in Fig. 6. $Z_{c1} = \rho c/S_1$ and $Z_{c2} = \rho c/S_2$ are the respective characteristic impedances of the front and back cavities, with $S_1 = \pi d_1^2/4$ and $S_2 = \pi d_2^2/4$ the cross sections of the back and front cavities (d_1 and d_2 are the respective diameters).

The characteristic dimensions of the sensor are given in Table II. With these dimensions, the first cut-off frequency of non-planar mode in the sensor is 11 kHz in the back cavity. Frequencies leading to singular values (0 or 1/0) of one parameter are given in Table II. The lower frequency, corresponding to $\tan(kL_2) = 0$, is 6.5 kHz, which can be considered as the theoretical limit of the sensor.

Only the relative sensitivity of the sensors does not depend on the geometrical dimensions of the sensor. So, these geometrical dimensions being accurately measured, complex functions β and δ are known analytically and no calibration is needed for these parameters. Microphones are Panasonic electret microphones (shown as mic 1 and mic 2 in Fig. 6) and are chosen so that their relative sensitivity is close to unity.

B. Calibration

As functions β and δ are analytically known, a calibration with a single load is sufficient to determine parameter K and, consequently, the relative sensitivity of the microphones. However, a complete calibration with three loads was performed in order to verify the accuracy of Eq. (15). Following Dickens *et al.*,[20] the three loads are non-resonant loads.

First is the radiation impedance of the output of the sensor, which can be calculated according to Nederveen and coworkers[21] (see also Ref. 9). The second load is a long pipe (50 m) whose input impedance can be assumed to be close to its characteristic impedance $Z = \rho c/S$, with $S = \pi d^2/4$ the cross sections of the pipe (d its diameter). The third load is a rigid metallic plate, which can be assumed to be infinite. The three transfer functions measured with these three loads allow the determination of the three complex functions K, β, and δ.

Analytical results for δ and β are accurate enough so that a calibration with a single load is sufficient [Figs. 7(a) and 7(b)]. The rigid plate is chosen to obtain the best signal to noise ratio. The relative sensitivity of the microphones (amplitude and phase) is fitted with a complex analytical function. The accuracy of the relative sensitivity is estimated to be $\pm1\%$ (0.1 dB) for the amplitude, and $\pm0.5°$ for the phase angle [Fig. 7(c)].

TABLE II. Characteristic dimensions of the set-up (refer to Fig. 6) and associated cut-off frequencies [refer to Eq. (15)].

Dimension (mm)	Cut-off frequency (kHz)
$L_1 = 21.4$	$\frac{c}{2L_1} = 8.0$
$L_2 = 13.0$	$\frac{c}{4L_2} = 6.6$
$L_1'' = 10.0$	$\frac{c}{4L_1''} = 8.6$
$L_2'' = 6.0$	$\frac{c}{4L_2''} = 14$
$d_1 = 18.0$	$\frac{1.84c}{\pi d_1} = 11$
$d_2 = 16.0$	$\frac{1.84c}{\pi d_2} = 12.5$

C. Signal processing

The transfer function between the two microphones is measured *via* a NI acquisition card and LABVIEW software developed by CTTM. Any type of signal can be used with this sensor. In the present experiment we chose to use a logarithmic chirp of 5 s duration leading to a frequency resolution of 0.2 Hz. When measuring the frequency response from 20 to 4000 Hz, it appears that the non-linear harmonic distortion on the sharpest resonances leads to a kind of ripple around frequencies that are multiples of these resonance frequencies. So, in order to limit these distortion effects and for a better signal to noise ratio, the impedance measurements of the trumpets are performed in three frequency bands: 20–500, 500–1000, and 1000–4000 Hz.

D. Accuracy

The relative accuracy depends on the accuracy of the functions K, β, and δ and also on the value of the impedance. In accordance with the small dimensions of the front and back cavities, the error of the impedance minima and their frequencies can be shown to depend mainly on the error of β; whereas the error of the impedance maxima and their frequencies may depend on the error of δ. When looking at the resonance frequencies of wind instruments, the error of δ can be expressed as a length correction error, setting $\delta = j\tan(kL_2)/Z_{c2}$. An error ΔL_2 on L_2 is equivalent to an error $\Delta L = (S_2/S)\Delta L_2$ on the length of the pipe where S is the cross section of the pipe. This results in a relative error of the resonance frequencies $\Delta f/f = -\Delta L/L$, where L is the equivalent length of the pipe. With an estimated uncertainty $\Delta L_2 = 1$ mm and a pipe of 2 m long and of diameter 8 mm against 16 mm for the sensor output, this leads to $\Delta f/f = 0.2\%$, which is 3 cents, and is not completely negligible. This error has to be compared with the error induced by an error in the temperature. An error of 1°C on the temperature leads to an error of 0.16%, which is also

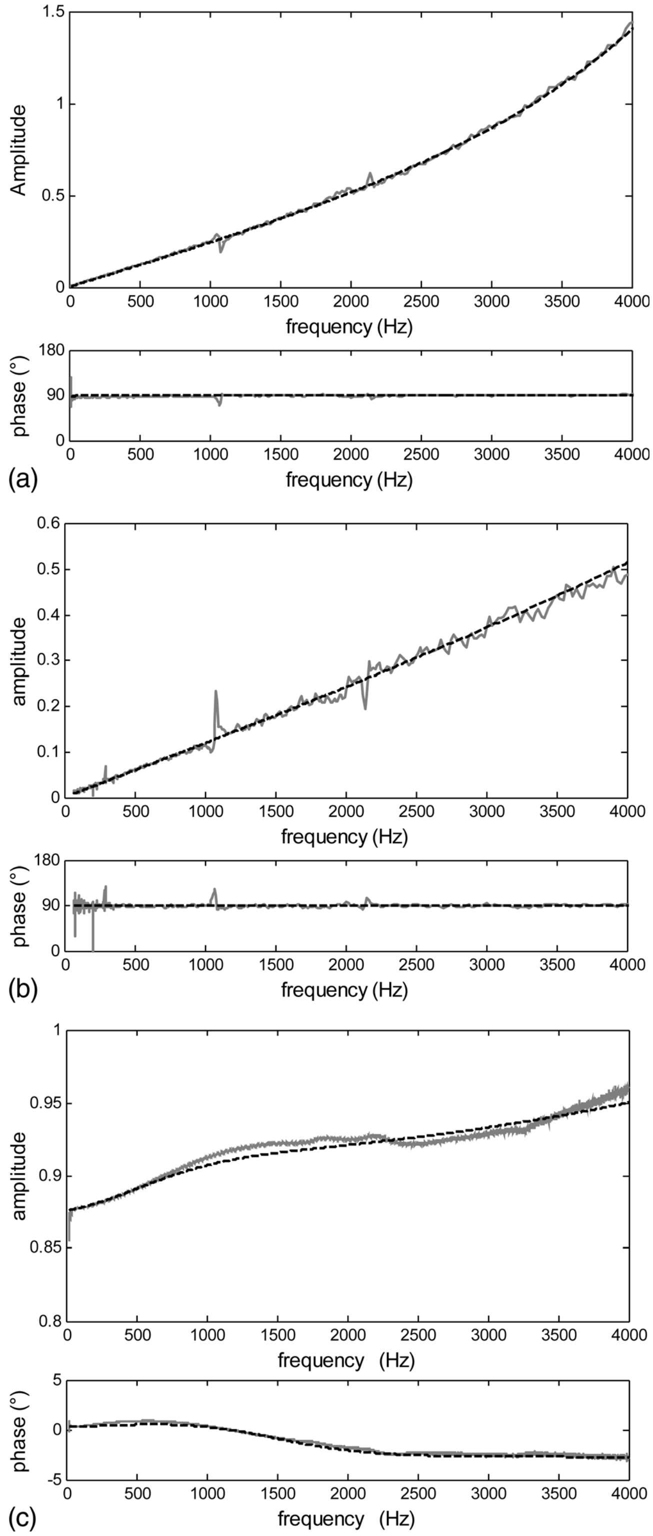

FIG. 7. Measurement and analytical model for amplitude and phase angle: (a) calibration parameter δ/Z_{c2}; (b) calibration parameter $\beta\ Z_{c2}$; (c) microphones sensitivity ratio. Gray line: measurement; Black dotted line: analytical model.

approximately 3 cents. When determining the resonance frequencies, an error is also induced by the experimental determination of the maxima of the input impedance. This error depends on the quality factor of the resonance. The resonance frequencies are determined by using a peak fitting technique using a least square method. The resulting error depends more

TABLE III. Measured trumpet resonance frequencies at 18.5°C, in hertz, with wall damping.

Acoustic resonance mode	Macaluso trumpet	Generic trumpet
1	82.75	81.70
2	231.1	229.4
3	346.3	343.0
4	460.0	458.9
5	574.3	581.6
6	689.9	700.9
7	802.4	816.2
8	916.9	934.0
9	1033.7	1045.7
10	1147.2	1153.3
11	1261.2	1262.1
12	1377.8	1372.2
13	1492.9	1485.2
14	1600.0	1605.5
15	1723	1722
16	1842	1823

on the quality factor of the resonances than on the frequency step of the Fourier transform, which is 0.2 Hz in the present experiment. The error in the resonance frequency is estimated to be 0.1 Hz for the second peak (230 Hz). This corresponds to a relative error of 0.05%, which is less than 1 cent and which can be considered valid for the first ten peaks. Finally, the absolute uncertainty of the resonance frequencies is estimated to be 0.3%, which is 5 cents. However, when comparing different resonances of the same instrument, the relative error is estimated to be ±3 cents, as systematic errors such as those due to temperature or sensor geometry are partly compensated.

VIII. MEASUREMENT COMPARISONS

A. Two trumpets

The resonance frequencies and the input impedance of two straight trumpets are measured. The mouthpiece, receiver, leadpipe, and tubing are unchanged in both trumpets, and only the bell wall shape differs in the two trumpets. The first trumpet includes a bell whose shape has been corrected to attain near-perfect harmonicity (Macaluso trumpet), whereas the second trumpet includes a commercially used generic bell of unknown shape (Generic trumpet).

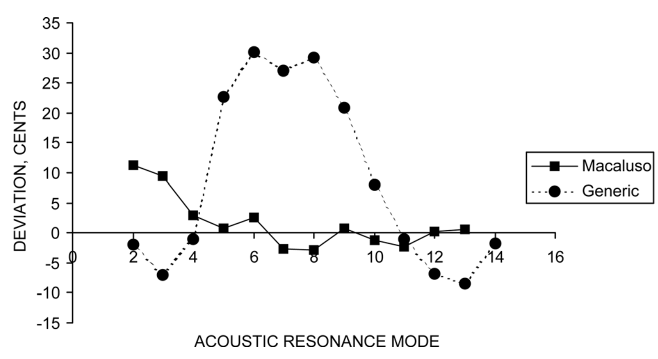

FIG. 8. Measured harmonicity deviations for two trumpets (damped tests).

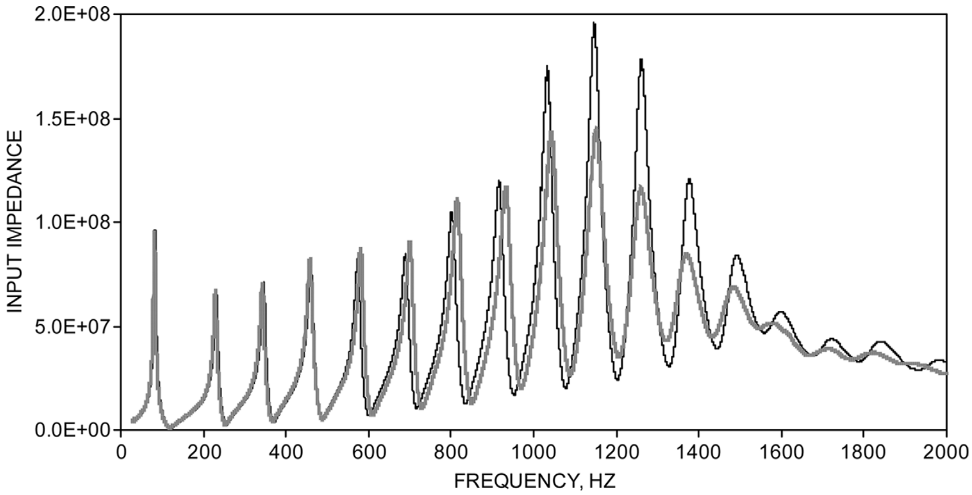

FIG. 9. Measured amplitude of input impedance for two trumpets. Black line: Macaluso trumpet; Gray line: Generic trumpet.

Some unexpected peaks in input impedance are visible at various frequencies. These peaks are interpreted as input impedance perturbation due to wall vibrations, similar to what has been observed by Nief *et al*.[22] To validate this explanation, a second measurement was made in which the bell was gently wrapped in a light plastic film (damped). The results show that wall vibrations are highly diminished by the plastic film and that such wall vibrations do not significantly affect the acoustic resonance frequencies. The acoustic resonance frequencies for the two trumpets, with wall damping, are given in Table III.

The measured harmonicity deviations (from the damped tests) for acoustic resonance modes 2–13 (the simulation model limits of accuracy) are shown in Fig. 8. The Macaluso and Generic trumpets both have large deviations above resonance 13.

The principal result is that the maximum harmonicity deviation for the Macaluso trumpet is much smaller (14 cents between 2nd and 8th peaks) than for the Generic trumpet (39 cents between 6th and 13th peaks). Note that the harmonicity deviations of the Generic trumpet are quite similar to those of a trumpet using a BH bell. See Fig. 5. This confirms that bell shapes similar to the BH bell shape are used commercially. Benade[1] stated that, “quite passable instruments are made from them.” The measured harmonicity deviations for the Macaluso trumpet are primarily a result of limitations in the simulation model and measurement error.

For acoustic resonance modes 2–13, the rms harmonicity deviation for the Macaluso trumpet is 5 cents, and it is 18 cents for the Generic trumpet. However, considering the estimated measurement uncertainty of ± 3 cents, each of those deviations is somewhat over-stated. Thus, from the point of view of rms harmonicity deviation, the superiority of the Macaluso trumpet throughout a practical musical range is demonstrated.

The comparison of the input impedances of both trumpets, shown in Fig. 9, emphasizes a clear difference in the two trumpets, up to the cut-off frequency. Residual peaks are visible for the Macaluso trumpet indicating that standing waves are still present in the trumpet. In other words, the reflection coefficient of the Macaluso trumpet is larger than that of the Generic trumpet. This may have an influence on the radiated sound, however, that is difficult to predict. The amplitudes of peaks 9–13 are larger for the Macaluso trumpet, by 2 dB (and greater).

B. Calculations

Measured and calculated harmonicity deviations for the Macaluso trumpet are shown in Fig. 10. For modes 2–13, the rms deviation between measurement and calculation is 3 cents. For modes 4–13, the rms deviation between measurement and calculation is 1 cent. The maximum deviations between measurement and calculation for modes 2 and 3 are somewhat larger (between 6 and 8 cents).

The plane wave approximation could be responsible for the mode 2 and 3 deviations. However, in principle, there is no reason that the relative error might be larger for such low frequency resonances. It is not excluded that unexplained measurements errors or small geometrical differences between the calculation model and the test instrument may be responsible for part of this deviation.

Figure 11 shows the calculated and measured input impedance for the Macaluso trumpet. There is some small difference in the amplitudes of the peaks primarily at higher frequencies. This difference is due to the plane wave approximation and the radiation model used. Such approximations do not ensure good impedance matching with the surroundings, as in actuality. In other words, the reflection coefficient in the calculation model of the bell remains larger than it is in reality.

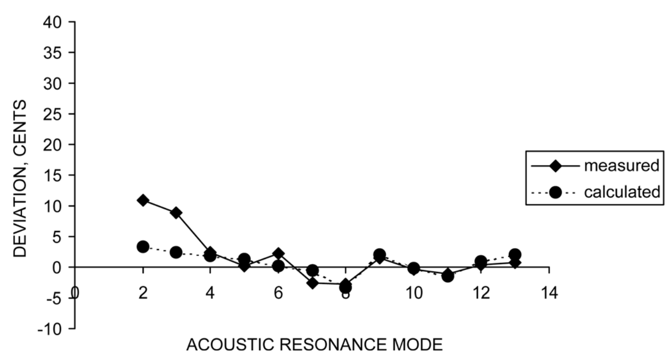

FIG. 10. Measured and calculated harmonicity deviations for Macaluso trumpet.

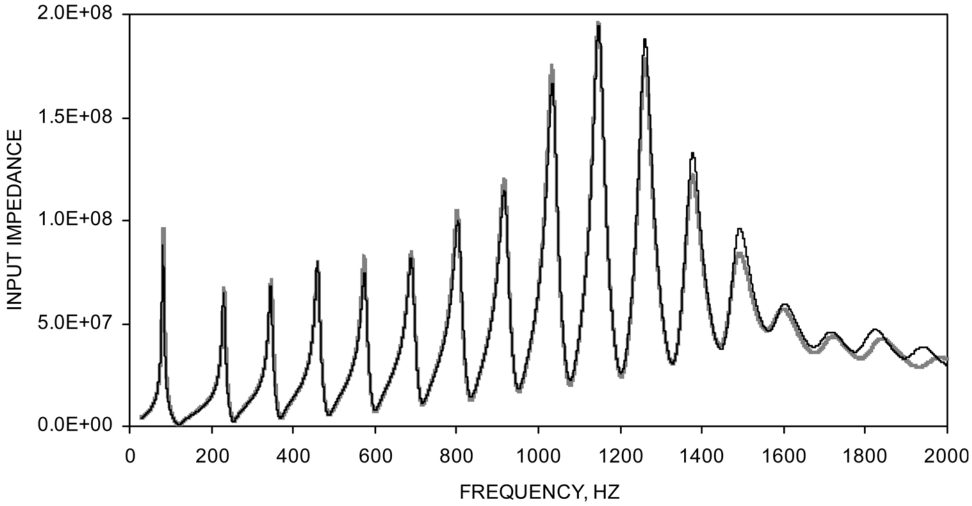

FIG. 11. Measured and calculated amplitude of input impedance for Macaluso trumpet. Black line: calculation; Gray line: measurement.

IX. CONCLUSION

A trumpet designed and built in accordance with the methodology presented is tested over a frequency range of 20–4000 Hz. The laboratory techniques for the determination of resonance frequencies and input impedance are new. The relative error is estimated to be ±3 cents. Over a musically useful range of acoustic resonance modes 2–13 (233–1515 Hz), the Macaluso trumpet is found to have a rms harmonicity deviation of 5 cents. Such a frequency range corresponds to that of the simulation model used. The same air column, with the bell replaced by a commercially used generic trumpet bell, is tested using the same means and is found to have rms deviation from harmonicity of 18 cents. However, considering the estimated measurement uncertainty, each of those deviations is somewhat over-stated. For that same range of resonances, the rms deviation between measured and theoretical calculated resonance frequencies for the Macaluso trumpet is 3 cents. The greatest deviations of measurements from calculation are for the second and third resonances. Such deviations are due to inadequacies in the simulation model used and possible unexplained measurement errors.

Based upon the measured results over a musically useful range, the harmonicity of the Macaluso trumpet is significantly better than that of the Generic trumpet. Over a musically useful range, the accord between theory and measurement for the Macaluso trumpet is good.

Since Benade,[1] it is generally acknowledged that near-perfect harmonicity is one of the conditions required for a good brass wind instrument. However, more recently, some musical acoustics researchers[23] have questioned whether harmonicity was essential. The degree of harmonicity that may be required is addressed by Benade's statement: "The more resonances that are present to cooperate and the more accurately these are aligned, the easier it is to play the notes."

Producing a trumpet with near-perfect harmonicity is an important first step in providing answers to harmonicity questions. To determine whether near-perfect harmonicity does affect the playing and sound qualities of a brass wind instrument requires that play testing must follow this work.

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